Error Probability of Equal Gain Combining for Wireless Digital Communication Systems in an Interference-limited Non-Gaussian Fading Channel

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Abstract—The performance evaluation of wireless digital communication systems employing equal gain combining (EGC) diversity technique at the receiver is investigated with the presence of co-channel interference in a non-Gaussian multipath fading environment. The closed-form expression of error probability is then derived in the case of both identical and distinct-powered interferers.

I. INTRODUCTION

In wireless communication systems, the transmitted signal is mainly distorted by multipath fading and co-channel interference. Several techniques such as antenna diversity, adaptive arrays, equalization, etc. have been proposed to combat the effect of multipath fading and reduce the co-channel interference. The most popular diversity techniques employed in wireless communication systems are equal gain combining (EGC), maximal ratio combining (MRC) and optimum combining (OC). The Optimum combining schemes give the outstanding performance by maximizing the signal-to-interference-plus-noise ratio (SINR), however the implementation is complex and in some environments the performance is in substantially different from the performance of MRC [1]. Although, the MRC maximizes the signal-to-noise ratio (SNR) [2], this combining technique needs the knowledge of amplitude and phase of the signals to maximize the signaling power. Then in many practical interests, the EGC is often used as a less complex alternative to the MRC, and also has a closed performance to the MRC when the estimation of fading channel amplitudes cannot be reliably accurate.

In wireless communication systems the multipath fading channel is generally modeled by the envelope distribution of Gaussian quadrature components such as Rice or Rayleigh distribution which are based on the assumption of infinite randomly scattering signals at the received antenna yielding the use of central limit theorem (CLT) [1]. However, in many practical environments the CLT may not be applied and the Gaussian processes fail to describe the phenomenon of wave propagation. Consequently, the conventional receiver model based on the Gaussian quadrature components can be far from its optimum performance. There are many non-Gaussian processes proposed in literatures for describing the multipath fading channel such as Nakagami, Wiellull, etc.[1]. Those models are based on empirical data and lack of the statistical properties of corresponding quadrature components. One non-Gaussian random process namely spherically invariant random process (SIRP) which generally models cluster in radar application is recently proved to model fading channel statistics [3]. This process is of interest mainly due to many Gaussian-liked properties and also has the Gaussian random process as a special case [3]. In this paper the performance analysis of wireless digital communication is derived in a currently practical environment that has a small cell size (Micro or Pico cells) and a lot of users or co-channel interference called interference-limited environment such as CDMA or Bluetooth personal communication system (PAN). The performance analysis of systems in such environment is only derived in literatures in Nakagami fading channel with equal-power interferers [4] or in Rayleigh-faded distribution [5]. Moreover to relax the assumption of CLT and Gaussian process, the fading channel statistics is assumed to be an SIRP. The closed-form expression of error probability is then derived for validate the system performance.

II. SYSTEM MODEL

The received signal at the output of an N-element antenna array in an interference-limited environment can be written by

\[ r(t) = \sqrt{P_s} s_i(t) \mathbf{a}_i + \sum_{i=1}^{L} \sqrt{P_i} s_i(t) \mathbf{a}_i, \]

where \( s_i(t) \), \( i = 0,1,2,3, ..., L \) are the transmitting signals, \( P_s \) and \( P_i \) are the power of desired and interfering signals and...
the vectors  \( \mathbf{A}_i = \begin{bmatrix} a_{i,0} & a_{i,1} & a_{i,2} & \cdots & a_{i,N} \end{bmatrix}^H \), \( i=0,1,2,3,\ldots,L \) are the \( N \)-dimensional direction of arrivals of the desired (\( i=0 \)) and the \( i \)th interfering signals and \( L \) is the number of interfering signals, respectively. The superscript \( H \) denotes the Hermitian transpose of the complex vector. We assume further that each component of the vectors \( \mathbf{A}_i \) follows a complex spherically invariant random vector (SIRV) with mean \( \mu \) and covariance matrix \( \mathbf{M} \). According to the representation theorem [3], the SIRV vector \( \mathbf{A}_i \) can be written as the product of a positive-valued random variable \( u_i \) and a zero mean complex Gaussian random \( \mathbf{Z}_i \), i.e.,

\[
\mathbf{A}_i = u_i \mathbf{Z}_i, \tag{2}
\]

where \( u_i \) and \( \mathbf{Z}_i \) are assumed to be independent for \( i = 0,1,2,3,\ldots,L \). Therefore, the probability density function (pdf), \( f_{\mathbf{A}_i}(\mathbf{A}) \) of \( \mathbf{A}_i \) can be given by the product of marginal pdf of \( u_i \) and \( \mathbf{Z}_i \), i.e., \( f_{\mathbf{A}_i}(\mathbf{A}) = f_{u_i}(u_i) f_{\mathbf{Z}_i}(z) \). It can be easily shown that for a given Gaussian random vectors, \( \mathbf{Z}_i \) the statistical properties of an SIRP can be completely characterized by the knowledge of the pdf of \( u_i \). Thus the pdf of \( u_i \) in (2) is so called characteristic pdf of an SIRP.

Note that for \( f_{u_i}(u_i) = \delta(u_i - 1) \) the non-Gaussian random vector \( \mathbf{A}_i \) reduces to the Gaussian random vector, where \( \delta(t) \) is a unit impulse function.

When EGC is employed at the received antenna array, the output SIR, \( \gamma \) may be written as

\[
\gamma = \frac{w^H \mathbf{A}_i \mathbf{A}_i^H w}{w^H \sum_{l=1}^{L} \mathbf{A}_l \mathbf{A}_l^H w}, \tag{3}
\]

where the outputs of all branches are co-phased and equally weighted by \( w = \begin{bmatrix} e^{j\theta_1} & \cdots & e^{j\theta_N} \end{bmatrix}^T \). The output SIR at the receiver employing EGC can then be expressed as [5]

\[
\gamma = P_e \left( \frac{\sum_{j=1}^{N} a_{0,j}}{\sum_{j=1}^{N} P_e \left( \frac{\sum_{j=1}^{N} a_{i,j}}{w} \right)^2} \right). \tag{4}
\]

Note that \( a_{i,j} \) for \( i = 0, 1, 2,\ldots, L \) and \( j = 1, 2, 3,\ldots, N \) are spherically invariant distributed, i.e. \( a_{i,j} = u_i z_{i,j} \) then (4) can be shown to be given by

\[
\gamma = u_0^2 \left( \frac{\sum_{j=1}^{N} z_{0,j}}{\sum_{j=1}^{N} \frac{u_i^2}{\Lambda_i} \sum_{j=1}^{N} z_{i,j}} \right)^2, \tag{5}
\]

where the parameter \( \Lambda_i = P_i / P_i \) for \( i=1,2,3,\ldots,L \) is the power ratio of desired to the \( i \)th interfering signals. Assuming that the fading environment is such that the characteristic pdf \( u_i \), corresponding to faded interferers are strongly correlated, i.e., all the entire interfering signals are modulated by the same positive random variable \( u_i = u \) which is independent of \( u_0 \) that modulates the desired signal. The output SIR is then written as

\[
\gamma = r \left( \frac{\sum_{j=1}^{N} z_{0,j}}{\sum_{j=1}^{N} \frac{1}{\Lambda_i} \sum_{j=1}^{N} z_{i,j}} \right)^2, \tag{6}
\]

where \( r = u_0^2 / u_i^2 \). It can be shown that the output SIR in (6) is easily evaluated by the product of a positive random variable, \( r \) and the output SIR of a Gaussian fading channel.

### III. PERFORMANCE ANALYSIS

In digital cellular communication systems, the probability of error is an important tool for measuring the system performance. It can be shown that the conditional average probability of error in interference-limited multipath fading channel can be written as [6]

\[
\overline{P_e} = \int_{0}^{\gamma} P_e(\gamma) f_{\gamma}(\gamma) d\gamma, \tag{7}
\]

where \( P_e(\gamma) \) is the conditional bit error probability for a given value of \( \gamma \), depending on types of detection scheme employed at the receiver and \( f_{\gamma}(\gamma|r) \) is the conditional pdf of the output SIR for a given \( r \) and the combining of diversity signals can be done either coherently or non-coherently. When the receiver employs the binary phase shift keying (BPSK), the conditional probability of error is then given by [6]

\[
P_e(\gamma) = \Gamma(b, \gamma) / 2\Gamma(b), \tag{8}
\]

where \( \Gamma(x, y) \) stands for the incomplete gamma function [7]. The coefficient \( b \) indicates the detection mechanism, i.e.,

\[
b = \begin{cases} 
1/2, & \text{for coherent}, \\
1, & \text{for noncoherent}, 
\end{cases} \tag{9}
\]

respectively [6]. Therefore, the average probability of error can be evaluated by
where $P_o(t|r)$ is the conditional outage probability of a wireless communication system employing EGC diversity scheme. There are many choices of SIRP that are generally used in literatures. However, the SIRP that has caught attention is the student-t distribution because it includes Gaussian, and Cauchy distribution as special cases.

The choices of the characteristic pdf correspond to assuming that the random matrix $\Lambda_i$ follows a student-t-distributed is used in the analysis, which the characteristic pdf is given by [3]

$$f_{u_i}(u_i) = \frac{2\nu^\nu \exp[-\nu/u_i^2]}{u_i^{2\nu+1}\Gamma(\nu)},$$

(11)

where $\nu$ denotes the degree of freedom (DOF) of $u_i$. This pdf can reduce to Cauchy when the DOF equals $1/2$ and tends to Gaussian distribution when the DOF equals $\infty$ [7]. The pdf of $r_i = u_0^2/u_i^2$ then can be obtained by

$$f_{r_i}(r_i) = \frac{\Gamma(2\nu)r_i^{\nu-1}}{\Gamma(\nu)(1+r_i)^{\nu}}.$$

(12)

Thus for student-t SIRP, substituting [8] in (10), yields

\[
\overline{P}_e = \frac{\nu}{2\Gamma(\nu)} \int_0^\infty \frac{\Gamma(b, \nu)}{2\Gamma(b)} f_r(r) \, dr
\]

(10)

\[
= \frac{1}{2\Gamma(b)} \int_0^\infty P_o(t|r) t^{\nu-1} e^{-t} \, dt
\]

after some numerical attempts, the outage probability for non-identical power interferers can be shown to be given by (16). Also for identical power interferers, substituting [8] in (10), gives

\[
\overline{P}_e = \frac{\nu}{2\Gamma(\nu)} \int_0^\infty \frac{\Gamma(b, \nu)}{2\Gamma(b)} f_r(r) \, dr
\]

(11)

\[
= \frac{1}{2\Gamma(b)} \int_0^\infty P_o(t|r) t^{\nu-1} e^{-t} \, dt
\]

IV. NUMERICAL RESULTS

Some numerical results are depicted and the results are compared with corresponding proposed mathematical analysis in order to check the accuracy of the derived equations. In Figure1. the bit error probability of EGC versus average SIR with $N$ diversity as a parameter, in the presence of eight non-identical power interferers ($L = 8$) is plotted. As expected the system performance is improved when the number of antenna ($N$) is increased. Figure2. provides the probability of error for a given number of antennas ($N = 2$) and number of equal power interferers ($L = 6$) with DOF ($\nu$).
as a parameter. It can be shown that when the DOF is very large ($\nu \rightarrow \infty$) the bit error probability tends to be as Rayleigh fading channel as provided in (15).

V. CONCLUSION
The performance measurement tool in wireless digital communication systems, i.e., probability of error is investigated in a non-Gaussian multipath fading environment when the receiver employs the EGC scheme. The non-Gaussian model called SIRP is chosen to model the non-Gaussian fading channel of wireless communication systems for general purpose. The numerical results show that this model gives a practical alternative approach for evaluating the system performance in non-Gaussian multipath fading channel that also includes Rayleigh fading model as a special case.

VI. REFERENCES

Figure 1. Probability of error of BPSK modulation scheme employing EGC versus average SIR with $N$ diversity as a parameter, in the presence of eight non-identical power interferes and SIRP fading channel.

Figure 2. Bit error probability of BPSK modulation scheme employing EGC technique with number of equal power interferes $L = 6$ and number of antenna arrays ($N = 2$) in Rayleigh fading ($m = 1$) comparison of SIRP fading channel when DOF as a parameter.
\[ P_k = \sum_{2 \leq i \leq k} \frac{\Gamma(2\nu)}{\pi^{2\nu}} \left[ A_k^{\nu} \Gamma(\nu) \Gamma(\nu + 1) \Gamma(3/2 - \nu)_2 F_2(2\nu, \nu, \nu - 1/2; -\Lambda) \right. \\
+ \frac{\Gamma(\nu - 1) \Gamma(\nu + 3/2) \Gamma(\nu + 1)}{-2\sqrt{\nu}} A_k^{\nu} \Gamma(\nu + 3/2, 3/2, -\nu + 5/2, 3/2; -\Lambda) \\
+ \left. A_k^{\nu} \Gamma^2(\nu) \Gamma^2(3/2 - \nu) \frac{\sqrt{\Lambda}}{\Gamma(2\nu)} \right] F_2(2\nu, \nu, \nu - 1/2; -\Lambda) \\
+ A_k^{\nu} \Gamma(\nu + 1) \Gamma(\nu + 1/2) F_2(2\nu, \nu + 1/2, \nu + 1/2; -\Lambda) \\
+ \frac{\pi A_k^{\nu} \Gamma(-\nu + 1/2) \Gamma(\nu + 1/2) \Gamma(\nu + 1)}{\Gamma(3/2)} F_2(2\nu, \nu + 1/2, \nu + 1/2; -\Lambda) \\
+ \left. \frac{\pi A_k^{\nu} \Gamma(-1) \Gamma(\nu + 1/2) \Gamma(\nu + 1/2) \Gamma(\nu + 1/2)}{\Gamma(3/2)} F_2(1/2) \right) \\
\left. \frac{\Gamma^2(\nu + 1/2) \Gamma(\nu + 3/2) \Gamma(\nu + 3/2, 3/2; -\Lambda) \right] F_2(2\nu, \nu, \nu - 1/2; -\Lambda) \\
+ A_k^{\nu} \Gamma(\nu + 1) \Gamma(\nu) \Gamma(-\nu + 3/2) F_2(2\nu, \nu, \nu - 1/2; -\Lambda) \right] \]

\[ P_\nu = \frac{1}{2} \left[ \frac{\Lambda}{2} \Gamma(\nu + L) \Gamma(\nu + L - 1) \Gamma(2\nu) \right. \\
- \left. \frac{1}{2} \Gamma(\nu + L, \nu + L + 1, \nu, \nu + 1/2; -\Lambda) \right] F_2(2\nu, \nu + L - 1, \nu, \nu + 1/2; -\Lambda) \\
- \frac{\Gamma(\nu + L) \Gamma(2\nu + L) \Gamma(3/2 - \nu + 1/2) \Gamma(L - \nu) \Gamma(L - \nu) \Gamma(b - 1)}{\Gamma(\nu + L - 1) \Gamma(2\nu + L + b) \Gamma(\nu + 1/2) \Gamma(L - \nu) \Gamma(b - 1)} \right] F_2(2\nu, \nu + 1, 2 - \nu, 2 - b; -\Lambda) \\
- \sqrt{\Lambda} \Gamma(\nu + 1/2) \Gamma(\nu + 1/2) \Gamma(L - \nu) \Gamma(-\nu - 1/2) \Gamma(L + 1/2) \right] F_2(2\nu, \nu + 1/2, \nu + 1; -\Lambda) \\
- \frac{\Gamma(\nu - 1/2) \Gamma(\nu - 3/2) \Gamma(L + 1/2 - b) \Gamma(L + 1/2)}{\Gamma(\nu + 1/2) \Gamma(2\nu) \Gamma(\nu + 1/2) \Gamma(L + 1/2)} \right] F_2(2\nu, \nu + 1/2, \nu + 1 - b, 1 - \Lambda) \\
+ \frac{(\Lambda^{\nu}) \Gamma(-\nu - 1/2) \Gamma(L - \nu) \Gamma(b - v) \Gamma(L + 1/2)}{\Gamma(\nu + \nu + L + 1/2, L + 1/2 - b, \nu - v + 1/2, b - v + 1/2; -\Lambda) \\
+ \frac{(\Lambda^{\nu}) \Gamma(-\nu - b) \Gamma(L - \nu) \Gamma(L - \nu) \Gamma(b - v) \Gamma(L + 1/2)}{\Gamma(\nu + \nu + L + 1/2, L + 1/2 - b, \nu - v + 1/2, b - v + 1/2; -\Lambda) \\
+ \sqrt{\Lambda} \Gamma(-\nu + 1/2) \Gamma(L - b + 1/2) \Gamma(\nu + 1/2 - b) \Gamma(L + 1/2) \right] F_2(2\nu, \nu + 1/2, \nu - L + 1, \nu - b + 1/2; -\Lambda) \\
- \frac{(\Lambda^{\nu}) \Gamma(-\nu - 1/2) \Gamma(L - b) \Gamma(L - b) \Gamma(b - v) \Gamma(L + 1/2)}{\Gamma(\nu + \nu + L + 1/2, b - v + 1/2, b - v + 1/2, L + 1/2; -\Lambda)} \\
- \frac{\sqrt{\Lambda} \Gamma(-\nu - 1/2) \Gamma(L - b + 1/2) \Gamma(\nu + 1/2 - b) \Gamma(L + 1/2) \right] F_2(2\nu, \nu + 1/2, \nu - L + 1, \nu - b + 1/2; -\Lambda)}{\Gamma(\nu + \nu + L + 1/2, L + 1/2 - b, \nu - v + 1/2, b - v + 1/2; -\Lambda)} \\
- \frac{\sqrt{\Lambda} \Gamma(-\nu + 1/2) \Gamma(L - b + 1/2) \Gamma(\nu + 1/2 - b) \Gamma(L + 1/2) \right] F_2(2\nu, \nu + 1/2, \nu - L + 1, \nu - b + 1/2; -\Lambda)}{\Gamma(\nu + \nu + L + 1/2, L + 1/2 - b, \nu - v + 1/2, b - v + 1/2; -\Lambda)} \]